Iridescence Education Center

Secondary Two Mathematics Final Mock Exam (June 2020)

COMPLETE SOLUTION

AND

MARKING SCHEME

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Secondary Two Mathematics — Mock-Exam (June 2020)

PAPER 1

Section A(1) (35 marks)

1. Solution

$$\frac{a-c}{c+b} = 2e$$

$a - c = 2ec + 2eb$ (1A)

$2ec + c = a - 2eb$

$c(2e+1) = a - 2eb$ (1A)

$c = \dfrac{a-2eb}{2e+1}$ (1A)

2. Solution

(a) (i) $12x^2 - 8xy = 4x(3x - 2y)$ (1A)

(ii) $9x^2 - 4y^2 = (3x)^2 - (2y)^2$ (1M)

$= (3x+2y)(3x-2y)$ (1A)

(b) $12x^2 - 8xy - 9x^2 + 4y^2 = (12x^2 - 8xy) - (9x^2 - 4y^2)$

$= 4x(3x-2y) - (3x+2y)(3x-2y)$ (1M for using (a))

$= (3x-2y)\left[4x - (3x+2y)\right]$

$= (3x-2y)(x-2y)$ (1A)

3. Solution

(a) $(x+y)^2 - (2x-y)^2 = (x+y+2x-y)(x+y-2x+y)$ (1M)

$= 3x(2y-x)$ (1A)

(b) $2x^2 - 8xy + 8y^2 = 2(x^2 - 4xy + 4y^2)$

$= 2\left[(x)^2 - 2(x)(2y) + (2y)^2\right]$ (1M)

$= 2(x-2y)^2$ (1A)

(c) $(x+y)^2 - (2x-y)^2 - 2x^2 + 8xy - 8y^2 = 3x(2y-x) - 2(x-2y)^2$ (1M for using (a) and (b))

$= 3x(2y-x) - 2(2y-x)^2$

$= (2y-x)\left[3x - 2(2y-x)\right]$

$= (2y-x)(5x-4y)$ (1A)

4. Solution

LHS $= (a-2b)^3 + (2a+5b)^3$

$= (a-2b+2a+5b)\left[(a-2b)^2 - (a-2b)(2a+5b) + (2a+5b)^2\right]$ (1M)

$= (3a+3b)(a^2-4ab+4b^2-2a^2-ab+10b^2+4a^2+20ab+25b^2)$ (2A)

$= 3(a+b)(3a^2+15ab+39b^2)$

$= 9(a+b)(a^2+5ab+13b^2)$

$=$ RHS (1A)

So, $(a-2b)^3 + (2a+5b)^3 \equiv 9(a+b)(a^2+5ab+13b^2)$.

5. Solution

$\frac{(3x^2y^{-1})^3}{9(x^{-5}y^3)^{-2}} = \frac{27x^6y^{-3}}{9x^{10}y^{-6}}$ (1A)

$= 3x^{6-10}y^{-3-(-6)}$ (1A)

$= 3x^{-4}y^3$

$= \frac{3y^3}{x^4}$ (1A)

6. Solution

$6.72 \times 10^4 = k(1.5)(8 \times 10^{24})(127 + 273)$ (1A)

$k = \frac{6.72 \times 10^4}{4800 \times 10^{24}}$

$k = \frac{1.4 \times 10^{-3} \times 10^4}{10^{24}}$ (1A)

$k = 1.4 \times 10^{-23}$ (1A)

7. Solution

Let x and y be the current ages of Andrew and Bruce respectively.

We have $\begin{cases} x = y + 15 & -(1) \\ x + 3 = 2(y+3) & -(2) \end{cases}$ (1M+2A)

Substitute (1) into (2), we have $y + 15 + 3 = 2y + 6$ (1M)

$y = 12$ (1A)

So, the current age of Bruce is 12 years old.

8. <u>Solution</u>

(a) (i) $\angle BDC = \angle BCD$ (base $\angle s$, isos. $\triangle$) (1M Either)

$\angle BDC + \angle BCD + \angle CBD = 180^\circ$ ($\angle$ sum of $\triangle$)

$\therefore \angle BDC = \dfrac{180^\circ - 36^\circ}{2}$

$= 72^\circ$ (1A)

(ii) $\angle ABC = \angle ACB = 72^\circ$ (base $\angle s$, isos. $\triangle$)

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$ ($\angle$ sum of $\triangle$)

$\therefore \angle BAC = 180^\circ - 2 \times 72^\circ$

$= 36^\circ$ (1A)

(b) $\angle BDC = \angle BAD + \angle ABD$ (ext. $\angle s$, of $\triangle$) (1M)

$\angle ABD = 72^\circ - 36^\circ$

$= 36^\circ$

$= \angle BAD$ (1A)

$\because \angle ABD = \angle BAD$

$\therefore \triangle ABD$ an isosceles triangle (sides opp. equal $\angle s$) (1A)

Section A(2)   (35 marks)

9. Solution

(a) $AC^2 = AB^2 + BC^2$ (Pyth. Theroem)   (1M)

$AB = \sqrt{18.5^2 - 15.3^2}$

$AB = 10.4 \text{ cm}$ (1A)

(b) $\dfrac{10.4}{x} = \cos 25°$ (1M)

$x = \dfrac{10.4}{\cos 25°}$

$x \approx 11.5 \text{ cm}$ (corr. to 3 sig. fig.)   (1A)

$\dfrac{15.3}{18.5} = \sin(\theta + 25°)$ (1M)

$\theta = \sin^{-1}\left(\dfrac{15.3}{18.5}\right) - 25°$

$\theta \approx 30.8°$ (corr. to 3 sig. fig.)   (1A)

10. Solution

Add a line $GE \, // \, BF$.

$\angle GEF + \angle BFE = 180°$ (int. $\angle s$, $GE \, // \, BF$)

$\angle GEF = 180° - 45°$

$\angle GEF = 135°$ (1A Either)

$\angle GEC + \angle CEF + \angle GEF = 360°$ ($\angle s$ at a p.t.)

$\angle GEC = 112°$

$\because \ \angle ADC = \angle GEC = 112°$

$\therefore \ AD \, // \, GE$ (corr. $\angle s$ are equal)   1

	No or Totally Incorrect Reasons	**Incomplete or Partially Correct Reasons**	**Complete and Totally Correct Reasons**
Score(s) Awarded	0 mark	1 or 2 mark(s)	3 marks

Criteria for Awarding Reason Scores

- Zero mark will be given if candidate CANNOT prove $AD \, // \, GE$ successfully.
- 2-3 marks will be given only if the reason for determining $AD \, // \, GE$ is correct.
- 1 mark will be given if less than 3 reasons are correctly stated.
- 3 marks will be given only if complete and totally correct reasons are shown.

11. Solution

$\frac{1}{a+b} : \frac{2}{b} = 2 : 5$

$\frac{b}{2(a+b)} = \frac{2}{5}$ (1M+1A)

$5b = 4a + 4b$

$b = 4a$

$\therefore\ a : b = 1 : 4$ (1A)

$\because\ a : b = 1 : 4$ and $b : c = 3 : 5$

$\therefore\ a : b : c = 3 : 12 : 20$ (1A)

12. Solution

$\text{LHS} = A(2x-3)^2 + B(x+3) + C$

$= 4Ax^2 - 12Ax + 9A + Bx + 3B + C$

$= 4Ax^2 + (B - 12A)x + 9A + 3B + C$ (1A)

$\text{RHS} = Cx^2 + (B-6)x + A$

$\because$ LHS ≡ RHS , by comparing the coefficients on both sides, we have

$B - 12A = B - 6 \implies A = \frac{1}{2}$ (1M Either)

$4A = C \implies C = 2$

$9A + 3B + C = A \implies 3B = -6$ (1A for at least one correct)

$\implies B = -2$ (2A for all correct)

13. Solution

$A0E_{(16)} = 10 \times 16^2 + 0 \times 16^1 + 14 \times 16^0$

$= 2574_{(10)}$ (1A)

By using short division, we have $2574_{(10)} = 101000001110_{(2)}$ (1M Either)

Hence, $A0E_{(16)} = 101000001110_{(2)}$ (1A)

14. Solution

For regular polygon, we have

$\frac{(n-2) \times 180°}{n} = 7 \times \frac{360°}{n}$ (∠ sum of polygon) , (sum of ext. ∠s of polygon) (1M+1A)

$\implies n = 16$ (1A)

15. Solution

$\because \cos\theta = \dfrac{8}{17}$ and θ is an acute angle

$\therefore \sin\theta = \dfrac{\sqrt{17^2 - 8^2}}{17}$ (1M for using Pyth. theorem)

$= \dfrac{15}{17}$ (1A)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{15}{8}$ (1A)

Hence, $\dfrac{1 + \tan\theta}{\sin\theta} = \dfrac{1 + \frac{15}{8}}{\frac{15}{17}}$

$= \dfrac{391}{120}$ (1A)

16. Solution

(a) (i) Maximum absolute error $= \dfrac{0.2 \text{ cm}}{2}$

$= 0.1 \text{ cm}$ (1A)

(ii) Let h be the actual height of the cylinder.

Volume of cylinder $= \pi \left(\dfrac{1.4}{2}\right)^2 h$

$= 0.49\pi h$

$\because$ Upper limit of $h = (2.6 + 0.1)$ cm

$= 2.7$ cm (1A for both)

Lower limit of $h = (2.6 - 0.1)$ cm

$= 2.5$ cm

$\therefore$ Upper limit of the actual volume $= 0.49\pi(2.7)$ (1M Either)

$= 1.323\pi \text{ cm}^3$

Lower limit of the actual volume $= 0.49\pi(2.5)$

$= 1.225\pi \text{ cm}^3$

Therefore, $1.225\pi \text{ cm}^3 \le \text{Actual Volume} < 1.323\pi \text{ cm}^3$ (1A)

(Do not accept '$\le$' for second inequality sign)

(b) The required percentage error $= \dfrac{\pi\left(\frac{1.4}{2}\right)^2(2.65) - \pi\left(\frac{1.4}{2}\right)^2(2.6)}{\pi\left(\frac{1.4}{2}\right)^2(2.65)} \times 100\%$ (1M)

$= 1.886792453\%$

$\approx 1.89\%$ (corr. to 3 sig. fig.) (1A)

Section B **(40 marks)**

17. Solution

(a) Consider $\begin{cases} 2x - y - 3t = 0 & -(1) \\ x + 3y + 2 = t & -(2) \end{cases}$

$3 \times (1) + (2) : 6x - 3y - 9t + x + 3y + 2 = y$ (1M)

$$7x + 2 = 10t$$

$$x = \frac{10t - 2}{7}$$

$$x = \frac{2(5t-1)}{7}$$ (1A)

** Remarks : PP-1 for not factorising $10t - 2$

Substitute $x = \frac{2(5t-1)}{7}$ into (1), we have

$$\frac{20t - 4}{7} - y - 3t = 0$$

$$y = -\frac{t+4}{7}$$ (OR equivalence) (1A)

(b) (i)

$$x + y = 3$$

$$\frac{2(5t-1)}{7} - \frac{t+4}{7} = 3$$ (1M for using (a))

$$\frac{10t - 2 - t - 4}{7} = 3$$

$$t = 3$$ (1A)

(ii) Solving $\begin{cases} 2x - y - 3t = 0 \\ x + 3y + 2 = t \\ x + y = 3 \end{cases}$,

From (a), we can reduce it into $\begin{cases} x = \frac{2(5t-1)}{7} \\ y = -\frac{t+4}{7} \\ x + y = 3 \end{cases}$

From (b)(i), we have $x + y = 3 \implies t = 3$,

Hence, we have

$$x = \frac{2[5(3) - 1]}{7} = 4$$ (1M for substituting $t = 3$)

And, $y = -\frac{(3) + 4}{7} = -1$.

Therefore, the solution is $x = 4$ and $y = -1$. (1A)

18. Solution

(a) $\angle BCD = \frac{(5-2)\times 180^\circ}{5}$ ($\angle$ sum of polygon) (1M for both)

$= 108^\circ$

$\angle FCD = \angle CFD = \angle FDC = 60^\circ$ (corr. $\angle$s , equil. $\triangle$)

$\therefore\ \angle BCF = 108^\circ - 60^\circ$

$= 48^\circ$

$\because\ BC = FC$,

$\therefore\ \angle BFC = \angle FBC$, (base $\angle s$, isos. $\triangle$) (1M)

As, $\angle BFC + \angle FBC + \angle BCF = 180^\circ$ ($\angle$ sum of $\triangle$)

Hence, $\angle BFC = \frac{180^\circ - 48^\circ}{2}$

$= 66^\circ$ (1A)

(b) Join AF and FE.

By similar argument of part (a), we have

$$\angle FDE = 48^\circ = \angle BCF.$$

$\because\ BC = FC = FD = ED$,

$\therefore\ \triangle BDF \cong \triangle EDF$. (SAS)

$\because\ AB = AE$, (given)

$AF = AF$ (common)

$FB = FE = AE$ (corr. sides of $\cong \triangle$s)

Therefore, $\triangle ABF \cong \triangle AEF$ (SSS) 1

	No or Totally Incorrect Reasons	Incomplete or Partially Correct Reasons	Complete and Totally Correct Reasons
Score(s) Awarded	0 mark	1 or 2 mark(s)	3 marks

Marking Criteria

- Zero mark will be given if candidate CANNOT prove $\triangle ABF \cong \triangle AEF$ successfully.
- 2-3 marks will be given only if the rules for proving $\triangle ABF \cong \triangle AEF$ is correct (eg. SSS).
- 1 mark will be given if no reasons are shown but candidate can demonstrate effective logical flow.
- 3 marks will be given only if complete and totally correct reasons are shown.

19. Solution

(a) (i)

Score	Class Boundaries	Class Mark	Frequency
21 - 30	20.5 - 30.5	25.5	2
31 - 40	30.5 - 40.5	35.5	2
41 - 50	40.5 - 50.5	45.5	5
51 - 60	50.5 - 60.5	55.5	10
61 - 70	60.5 - 70.5	65.5	8
71 - 80	70.5 - 80.5	75.5	7
81 - 90	80.5 - 90.5	85.5	4
91 - 100	90.5 - 100.5	95.5	2

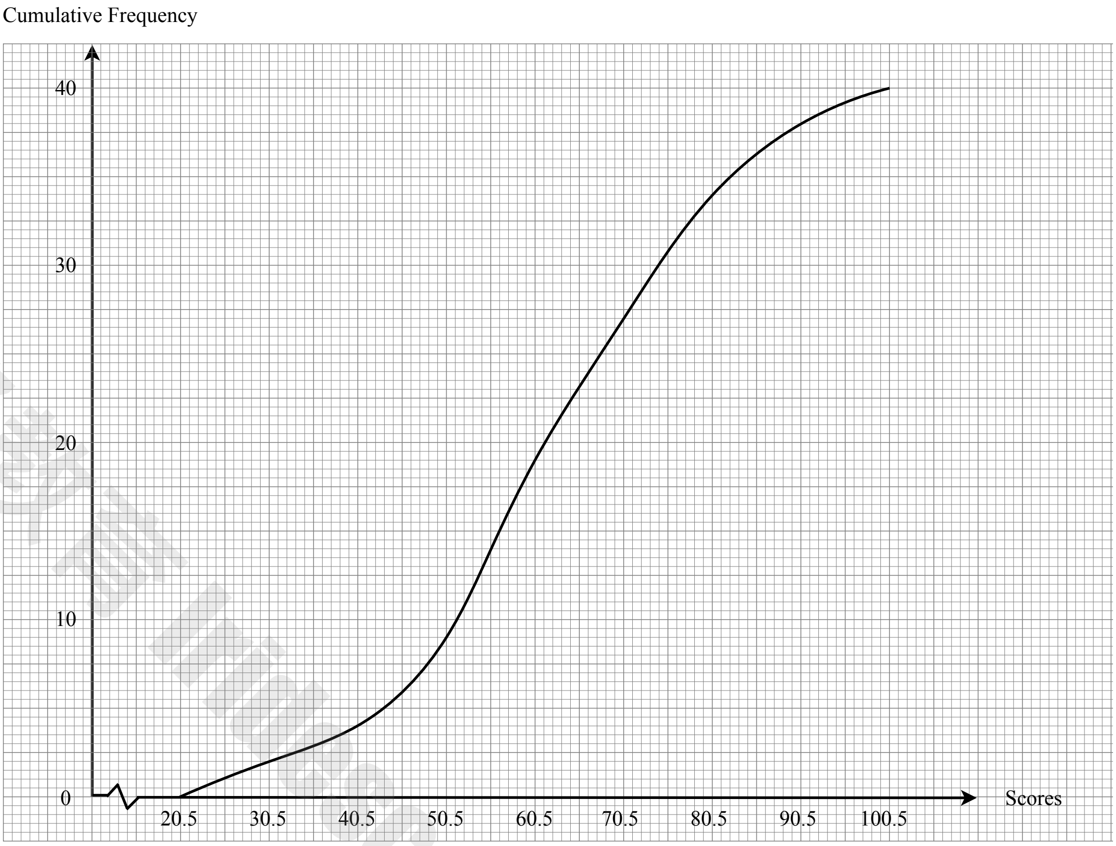
(1A for totally correct)

(Accept inequality signs ( ) $\leq x <$ ( ) for C.B.)

Score less than	Cumulative frequency
20.5	0
30.5	2
40.5	4
50.5	9
60.5	19
70.5	27
80.5	34
90.5	38
100.5	40

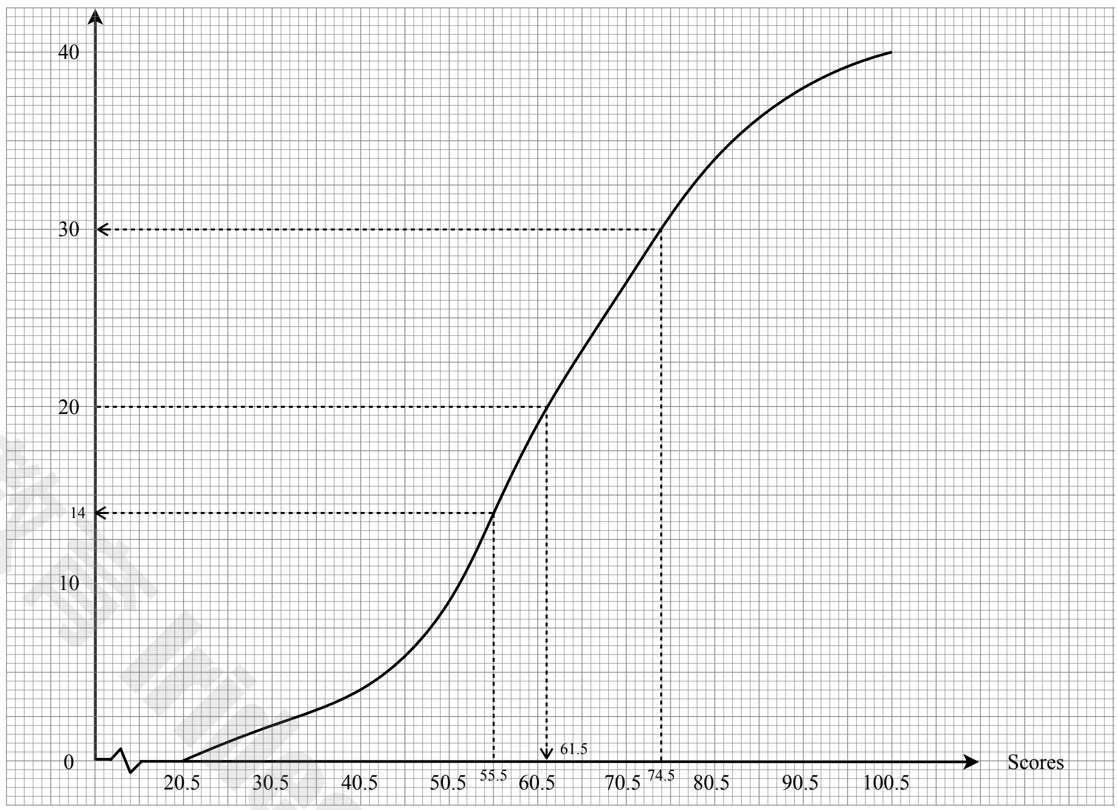
(1A for totally correct)

19. (a) (ii)



(1A for totally correct)

19. (b)



(i) Median = 61.5 scores (1A , accept answer ±0.5)

(ii) Number of students whose score < 55.5 is 14 .

Number of students whose score < 74.5 is 30 .

The required percentage $= \dfrac{30-14}{40} \times 100\,\%$ (1M)

$= 40\,\%$ (1A)

(For (b)(i) and (b)(ii), 0 marks will be given for no trace of using the cumulative frequency curve.)

(c) The 85th percentile = the 34th datum, (1A)

which lies on the class of 71-80 scores.

Since, Jonathan has obtained 82 scores > 80 scores. (1A)

Hence, he will be selected. (1A)

<u>Alternative Answer</u>

Since, Jonathan has obtained 82 scores.

His score belongs to the class of 81-90 scores. (1A)

There are at least $2+2+5+10+8+7=34$ students whose scores are lower than him.

As, $\dfrac{34}{40} \times 100\,\% = 85\,\%$

Therefore, Jonathan's score must lie within the top 15% . (1A)

Hence, he will be selected. (1A)

20. Solution

(a) (i) The required area $= \pi(8)^2 \times \dfrac{108^\circ}{360^\circ}$ (1M)

$= \dfrac{96}{5}\pi \text{ cm}^2$ (1A)

(ii) The required capacity $= 12 \times \dfrac{96}{5}\pi$ (1M)

$= \dfrac{1152}{5}\pi \text{ cm}^3$ (1A)

(iii) Perimeter of the base $= 2\pi(8) \times \dfrac{108^\circ}{360^\circ} + 8 \times 2$

$= \left(\dfrac{24}{5}\pi + 16\right) \text{cm}$

Hence, the required area $= \dfrac{96}{5}\pi - \pi\left(\dfrac{2}{2}\right)^2 + \left(\dfrac{24}{5}\pi + 16\right) \times 12$ (1M+1A)

$= \left(\dfrac{379}{5}\pi + 192\right) \text{cm}^2$ (1A)

(b) The volume of water inside the tank $= \dfrac{1152}{5}\pi - \pi\left(\dfrac{2}{2}\right)^2 \times 12$

$= \dfrac{1092}{5}\pi \text{ cm}^3$ (1A)

Let h be the required depth of water.

Consider $\dfrac{1092}{5}\pi = \dfrac{96}{5}\pi h$ (1M)

We, have $h = \dfrac{91}{8}$ cm (1A)

New total wet surface area $= \dfrac{96}{5}\pi + \left(\dfrac{24}{5}\pi + 16\right) \times \dfrac{91}{8}$

$= \left(\dfrac{369}{5}\pi + 182\right) \text{cm}^2$ (1A)

Hence, the required percentage change $= \dfrac{\frac{369}{5}\pi + 182 - \frac{379}{5}\pi - 192}{\frac{379}{5}\pi + 192} \times 100\%$

$= -3.785618817\%$

$\approx -3.79\%$ (1A)

21. Solution

(a) (i) Substitute $(3, -1)$ into $x + 2y = c$, we have

$$3 + 2(-1) = c$$

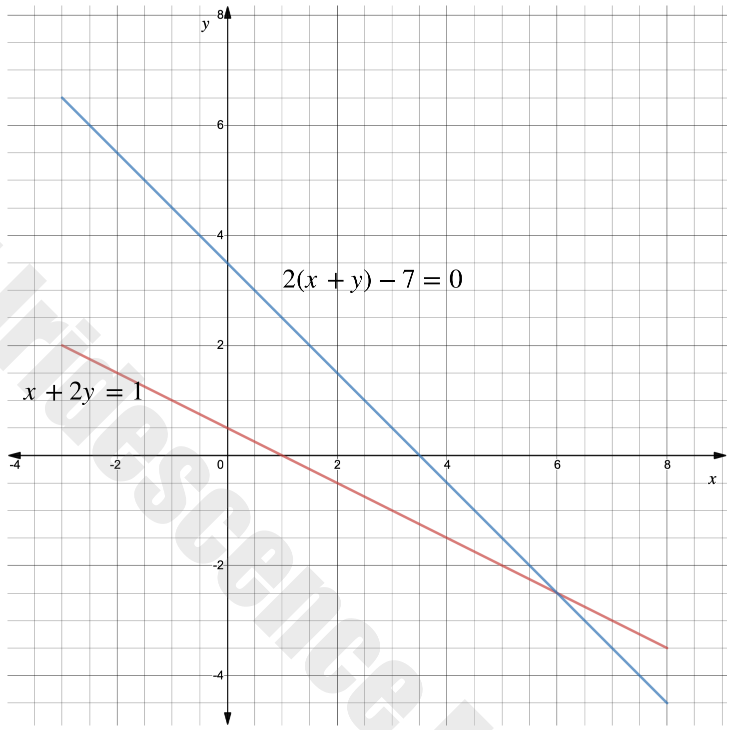
$$c = 1 \qquad \text{(1A)}$$

Substitute $c = 1$ and $(-1, k)$ into $x + 2y = c$, we have

$$-1 + 2k = 1$$

$$k = 1 \qquad \text{(1A)}$$

(ii)



(1A)

(b) By plotting $2(x + y) - 7 = 0$ in the same graph,

we can observe that the two lines intersect at $(6, -2.5)$. (1M)

Therefore, the required solutions are

$x = 6$ and $y = -2.5$. (1A for all correct)

— End of Paper —