COMPLETE SOLUTION

AND

MARKING SCHEME

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Secondary Three Mathematics — Mock-Exam (June 2020)

PAPER 1

Section A(1) (38 marks)

1. Solution

(a) $8u^3 - 1 = (2u)^3 - (1)^3$ (1M)

$= (2u - 1)(4u^2 + 2u + 1)$ (1A)

(b) $4u^2 + 18u - 10 = 2(2u^2 + 9u - 5)$

$= 2(2u - 1)(u + 5)$ (1A)

(c) $8u^3 - 1 - 4u^2 - 18u + 10$

$= (2u - 1)(4u^2 + 2u + 1) - 2(2u - 1)(u + 5)$ (1M)

$= (2u - 1)\left[4u^2 + 2u + 1 - 2(u + 5)\right]$

$= (2u - 1)(4u^2 - 9)$

$= (2u - 1)\left[(2u)^2 - 3^2\right]$ (1M)

$= (2u - 1)(2u - 3)(2u + 3)$ (1A)

2. Solution

Let x be the number of lemons originally in the basket.

$\frac{x - 4}{x + (x - 4)} = \frac{4}{9}$ (1M+1A)

$9x - 36 = 8x - 16$

$x = 20$

So, the required number is 20 . (1A)

3. Solution

(a) Required volume $= \frac{4}{3}\pi(5)^3 \times \frac{1}{8}$ (1M Either)

$= \frac{125}{6}\pi$ cm³ (1A)

(b) Required are $= 4\pi(5)^2 \times \frac{1}{8} + \pi(5)^2 \times \frac{1}{4} \times 3$ (1A for correct formula)

$= \frac{125}{4}\pi$ cm² (1A)

(PP-1 for not expressing answers in terms of π)

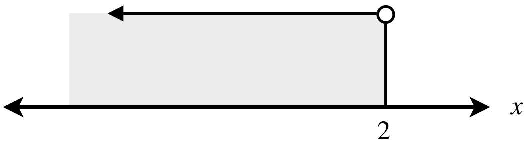
4. <u>Solution</u>

(a) $$\frac{2x+3}{5}-\frac{2(1+x)}{4}>-\frac{1}{10}$$

$8x+12-10(1+x)>-2$ (1M)

$-2x>-4$

$x<2$ (1A)



(1A)

(b) 0 and 1 (1A)

5. <u>Solution</u>

$$75000\left(1+\frac{r\,\%}{3}\right)^3-75000=45906$$ (1M + 1A)

$r=6$ (1A)

6. <u>Solution</u>

The required probability

$$=\frac{\pi(2)^2}{\pi\left(\frac{8}{2}\right)^2}$$ (1M)

$$=\frac{1}{4}$$ (1A)

7. Solution

$$\text{LHS} = \frac{1}{\tan^2(90^\circ - \theta)}\left[\frac{\cos\theta}{1-\sin^2(90^\circ - \theta)} - \sin(90^\circ - \theta)\right]$$

$$= \frac{\sin^2\theta}{\cos^2\theta}\left(\frac{\cos\theta}{1-\cos^2\theta} - \cos\theta\right) \qquad \text{(1M+1A)}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}\left(\frac{\cos\theta}{\sin^2\theta} - \cos\theta\right)$$

$$= \frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos\theta - \cos\theta\sin^2\theta}{\sin^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta}$$

$$= \cos\theta$$

$$\text{RHS} = \tan(90^\circ - \theta)\sin\theta$$

$$= \frac{\cos\theta}{\sin\theta}\cdot\sin\theta \qquad \text{(1A)}$$

$$= \cos\theta$$

$\because$ LHS = RHS

$$\therefore \frac{1}{\tan^2(90^\circ - \theta)}\left[\frac{\cos\theta}{1-\sin^2(90^\circ - \theta)} - \sin(90^\circ - \theta)\right] \equiv \tan(90^\circ - \theta)\sin\theta \qquad \text{(1)}$$

8. Solution

(a) Let the required angle be θ .

$$\tan\theta = \frac{1}{2.4}$$

$$\theta = 22.61986495^\circ$$

$$\approx 22.6^\circ \quad \text{(correct to 1 d.p.)}$$ (1A)

(b) (i) Horizontal distance $= 340\cos\theta$ (1M Either)

$$= \frac{4080}{13} \text{ m} \approx 313.8 \text{ m} \quad \text{(correct to 1 d.p.)}$$ (1A Both)

(ii) Vertical distance $= 340\sin\theta$

$$= \frac{1700}{13} \text{ m} \approx 130.8 \text{ m} \quad \text{(correct to 1 d.p.)}$$

(c) Let the required distance be x .

$$\frac{AB}{BC+x} = \tan 15^\circ$$ (1M)

$$x = \frac{\frac{1700}{13}}{\tan 15^\circ} - \frac{4080}{13}$$ (1A)

$$x = 174.1912595 \text{ m}$$

$$\approx 174.2 \text{ m} \quad \text{(correct to 1 d.p.)}$$ (1A)

9. Solution

(a) $\because$ Median $= 68$

$$\frac{67 + (a+60)}{2} = 68$$

$\therefore$ $a = 9$ (1A)

$\because$ Mean $= 71$

$$\frac{(90+b)+1752}{26} = 71$$

$\therefore$ $b = 4$ (1A)

(b) (i) $$\frac{42n + 71\times 26}{n+26} = 68$$ (1M)

$$n = 3$$ (1A)

(ii) Since $42\times 3 - 68 = 58$,

There must be at least 2 students whose scores are < 68 . (1A)

Thus, the median must decrease.

The claim is correct. (1A)

Section A(2) **(38 marks)**

10. Solution

(a) (i) $a = 72\cos(180° - 115°) + 58\cos 40°$

$= 74.85909255$ km

≈ 74.9 km (correct to 1 d.p.)

$b = 72\sin(180° - 115°)$

$= 65.25416067$ km

≈ 65.3 km (correct to 1 d.p.)

$c = 58\sin 40°$

$= 37.28168136$ km

≈ 37.3 km (correct to 1 d.p.)

(2M+2A)

— 1M for considering $180° - \theta$ and 1M for correct method for finding a

— 2A for all correct ; 1A for partial correct.

(ii) $(AC)^2 = a^2 + (b - c)^2$ (Pyth. Theorem)

$AC = 79.91460026$ km

≈ 79.9 km (1A)

(b) Let θ be the angle between the N-S axis and AC.

$\cos\theta = \dfrac{a}{AC}$ (1M)

$\theta = 20.48912846°$

$\approx 20.5°$

Hence, the required bearing is S20.5°E . (1A)

11. Solution

(a) (i) Coordinates of M

$$= \left(\frac{8+2}{2}, \frac{12+0}{2} \right) \quad \text{(Mid-p.t. Formula)} \quad \text{(1M)}$$

$$= (5,\ 6) \quad \text{(1A)}$$

(ii) $\because \quad m_L \times m_{AB} = -1$ (1M)

$$m_L \times \frac{12-0}{8-2} = -1$$

$$\therefore \quad m_L = -\frac{1}{2} \quad \text{(1A)}$$

(b) (i) Let the coordinates of $P = (x,\ 0)$ and $Q = (0,\ y)$.

$$m_{MP} = m_L$$

$$\frac{0-6}{x-5} = -\frac{1}{2} \quad \text{(1M Either)}$$

$$x = 17$$

$$m_{MQ} = m_L$$

$$\frac{y-6}{0-5} = -\frac{1}{2}$$

$$y = \frac{17}{2}$$

$$\therefore \ P = (17,\ 0),\ Q = \left(0,\ \frac{17}{2}\right) \quad \text{(1A for both correct)}$$

(ii)

$$PM = \sqrt{(17-5)^2 + (0-6)^2} \quad \text{(1M Either)}$$

$$= 6\sqrt{5} \text{ units}$$

$$MQ = \sqrt{(5-0)^2 + \left(6 - \frac{17}{2}\right)^2}$$

$$= \frac{5\sqrt{5}}{2} \text{ units}$$

$$\therefore \ PM : MQ = 12 : 5. \quad \text{(1A)}$$

12. <u>Solution</u>

(a) Let h be the height of the small vessel.

$\because \dfrac{h}{h+12} = \dfrac{8}{18}$ (1M)

$h = \dfrac{48}{5}$ m (1A)

$\therefore$ The required capacity $= \dfrac{1}{3}\pi\left(\dfrac{18}{2}\right)^2\left(\dfrac{48}{5}+12\right) - \dfrac{1}{3}\pi\left(\dfrac{8}{2}\right)^2\left(\dfrac{48}{5}\right)$ (1M)

$= 532\pi \text{ cm}^3$ (1A)

(b) Slant height of the small cone

$= \sqrt{4^2 + \left(\dfrac{48}{5}\right)^2}$ (Pyth. Theorem)

$= \dfrac{52}{5}$ cm

Slant height of the large cone

$= \sqrt{9^2 + \left(\dfrac{48}{5}+12\right)^2}$ (Pyth. Theorem)

$= \dfrac{117}{5}$ cm

The required area $= \pi(9)\left(\dfrac{117}{5}\right) - \pi(4)\left(\dfrac{52}{5}\right) + \pi(4)^2 + \pi(9)^2$ (1M+1A)

$= 266\pi \text{ cm}^2$ (1A)

13. Solution

(a) (i) Gradient of $AQ = \dfrac{350-150}{\frac{6\times 25000}{100}}$ (1M Either)

$= \dfrac{2}{15}$ (1A)

Gradient of $PQ = \dfrac{450-150}{\frac{7.2\times 25000}{100}}$

$= \dfrac{1}{6}$ (1A)

(ii) $\because$ Gradient of AQ < Gradient of PQ.

AQ is less steep than PQ.

$\therefore$ I don't agree (1A)

(b) Horizontal distance of $AP = \sqrt{7.2^2-6^2}\times\dfrac{25000}{100}$ (Pyth. Theorem) (1M)

≈ 994.9874371 m

The required angle of inclination $= \tan^{-1}\dfrac{450-350}{994.9874371}$ (1A)

$= 5.739170477°$

$\approx 5.74°$ (correct to 1 d.p.) (1A)

14. Solution

(a) $k = 1-0.26-0.17-0.12-0.06-0.03$

$= 0.36$

$1.57 = (-3)(0.36)+2(0.26)+h(0.17)+7(0.12)+8(0.06)+10(0.03)$

$h = 3$

(b) (i) Required probability

$= 0.36+0.26+0.17$

$= 0.79$

(ii) Required probability

$= 0.36+0.26+0.06+0.03$

$= 0.71$

(c) The new expected value of drawing a card

$= 1.57\times 3-1$

$= 3.71$

< 4

Therefore, the game is not favourable to play.

Section B **(24 marks)**

15. Solution

(a) (i) $\because \dfrac{a+6}{2}=2$ (Mid-p.t. Formula) (1M Either)

$\therefore \quad a=-2$ (1A Either)

$\because \dfrac{1+b}{2}=-1$ (Mid-p.t. Formula)

$\therefore \quad b=-3$

(ii) $m_{AB}=\dfrac{-3-1}{6-(-2)}=-\dfrac{1}{2}$

$\because \quad m_{AD}\times m_{AB}=-1$

$\dfrac{y-1}{c-(-2)}\times\left(-\dfrac{1}{2}\right)=-1$ (1A)

$\therefore \quad y=2c+5$ (1A)

(b) (i) $\because$ B and E are mid-p.t. of AC and DC respectively.

$\therefore$ $AD \,//\, BE$ (Mid-p.t. Theorem)

Hence, $BE \perp AC$. (1A)

(For this question, 0 mark for no/incorrect explanation despite correct answer.)

(ii) $AD=\sqrt{(c+2)^2+(2c+5-1)^2}$ (1M)

$=\sqrt{5c^2+20c+20}$

$=\sqrt{5(c+2)^2}$

$=\sqrt{5}(c+2)$ (1A)

$BE=\dfrac{1}{2}AD$ (Mid-p.t. Theorem) (1M)

$=\dfrac{\sqrt{5}}{2}(c+2)$

$AB=\sqrt{(-2-2)^2+(1+1)^2}$

$=2\sqrt{5}$ units

Thus, the area of $\triangle AEB=\dfrac{AB\times BE}{2}$

$=\dfrac{5(c+2)}{2}$ (1A)

15. (Continue)

(b) (iii) $AC = 2AB = 4\sqrt{5}$ units

The area of $\triangle ADC = \dfrac{AD \times AC}{2}$

$= 10(c + 2)$ (1A)

Thus, the required ratio is $= \dfrac{5}{2}(c + 2) : 10(c + 2)$ (1M)

$= 1 : 4$ (1A)

16. <u>Solution</u>

(a) As C is the in-centre of $\triangle PQR$, PC bisects $\angle RPQ$ and RC bisects $\angle PRQ$. (1A)

We have, $\angle QPR + \angle QRP + \angle PQR = 180°$ ($\angle$ sum of $\triangle$)

$$\angle QPR + \angle QRP = 180° - x$$

Then, consider $\angle PCR + \angle CPR + \angle CRP = 180°$ ($\angle$ sum of $\triangle$)

$$\angle PCR = 180° - \angle CPR - \angle CRP$$

$$= 180° - \frac{1}{2}\angle QPR - \frac{1}{2}\angle QRP$$

$$= 180° - \frac{1}{2}\left(\angle QPR + \angle QRP\right) \quad \text{(1M)}$$

$$= 180° - \frac{1}{2}(180° - x)$$

$$= 90° + \frac{x}{2} \quad \text{(1A)}$$

(b) (i) $\because \angle PCR = 135°$

$\therefore x = 90°$ (1A)

$\because m_{PQ} \times m_{QR} = -1$

$$\frac{k-(-8)}{-7-5} \times \frac{1-(-8)}{h-5} = -1 \quad \text{(1M)}$$

$\therefore h = \dfrac{3k+44}{4}$ (OR equivalent) (1A)

(ii) $\because h : k = 7 : 2$

$\therefore h = 14$ and $k = 4$ (1A)

Thus, $a = \sqrt{(4+8)^2 + (-7-5)^2}$

$= 12\sqrt{2}$ units

$b = \sqrt{(-8-1)^2 + (5-14)^2}$

$= 9\sqrt{2}$ units

$c = \sqrt{(-7-14)^2 + (4-1)^2}$

$= 15\sqrt{2}$ units

Marking scheme for finding the values of a, b and c,

— 1A for all correct ; 0A for partial correct or all incorrect.

16. (Continue)

(b) (iii) $A = \dfrac{ab}{2}$

$= \dfrac{12\sqrt{2} \times 9\sqrt{2}}{2}$

$= 108$ sq. units

Consider $\dfrac{a+b+c}{2} = \dfrac{A}{r}$

We have $\dfrac{12\sqrt{2} + 9\sqrt{2} + 15\sqrt{2}}{2} = \dfrac{108}{r}$ (1M)

$r = \dfrac{6}{\sqrt{2}}$

$r = 3\sqrt{2}$ units (1A)

(PP-1 for not rationalise the answer.)

(c) Orthocentre of $\triangle PQR = Q$.

So, points O and Q coincide. (1A)

$\because$ G is inscribed inside $\triangle PQR$.

$\therefore$ $CO > r$. (1A)

17. <u>Solution</u>

(a) (i) Yes.

$\because\ \angle EBC = \angle BCE$

$\therefore\ EB = EC$ (sides opp. equal $\angle s$) (1A)

$AB = BE = AF = FE$ (properties of rhombus)

Therefore, $FE = EC$.

Thus, E is the mid-point of FC. (1A)

(ii) $AB = EC$

$\because\ ED : DC = 2 : 1$, and

$ED : EC = 2 : 3$ } (1M)

$\therefore\ AB : ED = 3 : 2$. (1)

(iii) Let x be the length of DC and h be the height of $\triangle PED$.

We have $ED = 2x$ and $AB = FE = 3x$.

So, the height of $ABEF = \dfrac{3+2}{2}h$ (1M)

$= \dfrac{5}{2}h$

Area of $ABEF = 3x\left(\dfrac{5}{2}h\right)$ (1M Either)

$= \dfrac{15}{2}xh$ (1A)

Area of $\triangle PED = \dfrac{1}{2}(2x)(h)$

$= xh$ (1A)

Hence, we have $\dfrac{\text{Area of } ABEF}{\text{Area of } \triangle PED} = \dfrac{\frac{15}{2}xh}{xh}$

$= \dfrac{15}{2}$

Therefore, Area of $ABEF$: Area of $\triangle PED = 15 : 2$. (1A)

(b) Area of $\triangle BEC = \dfrac{1}{2} \times$ Area of $ABEF$

$= \dfrac{1}{2}a$ (1A)

From (a)(iii), we have Area of $\triangle PED = \dfrac{2}{15}a$ (1M)

Hence, the area of quadrilateral $BPDC = \dfrac{1}{2}a - \dfrac{2}{15}a$

$= \dfrac{11}{36}a$ (1A)

18. Solution

(a) (i) Arc $AB = 2\pi r \times \frac{216°}{360°}$

$= \frac{6}{5}\pi r$ cm (1A)

$\because\ 2\pi R = \text{Arc } AB$

$\therefore\ 2\pi R = \frac{6}{5}\pi r$ (1M)

$R = \frac{3r}{5}$ (1)

(ii) Consider $h^2 + R^2 = r^2$ (Pyth. Theorem)

$h = \sqrt{r^2 - \left(\frac{3}{5}r\right)^2}$ (1M)

$h = \sqrt{\frac{16}{25}r^2}$

$h = \frac{4}{5}r$ (1A)

(b) $V = \frac{1}{3}\pi R^2 h$

$= \frac{1}{3}\pi\left(\frac{3r}{5}\right)^2\left(\frac{4r}{5}\right)$ (1M)

$= \frac{12}{125}\pi r^3$ (1)

(c) Please refer to the next page.

18. (Continue)

(c) Since,

The volume of the ice-cream ball $= \frac{4}{3}\pi a^3$ (1A)

Consider $\left(\frac{h}{H}\right)^3 = \frac{\frac{12}{125}\pi r^3}{\frac{4}{3}\pi a^3}$ (1M)

$$= \frac{9}{125}\left(\frac{r}{a}\right)^3$$

$$= \frac{9}{125}\left(\sqrt[3]{24}\right)^3$$

Hence, $\frac{h}{H} = \frac{6}{5}$. (1A)

Since, $\frac{\text{Wet surface area}}{\text{Surface area of cone}} = \left(\frac{5}{6}\right)^2$ (1M)

$$= \frac{25}{36}$$

Hence, the percentage of the area of cone covered by the liquid ice-cream is

$= \frac{25}{36} \times 100\,\%$

$= 69.4\,\%$

$< 70\,\%$

Therefore, I don't agree. (1A)

Zero mark if there is no trace of logical explanations or arguments despite correct answer.

— End of Paper —